Exam. Code : 103205
Subject Code : 1217

B.A./B.Sc. Semester—V
MATHEMATICS
(Linear Algebra)
Paper—II

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt any **FIVE** questions in all choosing at least **TWO** from each Section.

SECTION—A

I. (a) Define Ring. Give an example of :

(i) a commutative ring without unity

(ii) a non-commutative ring with unity.

(b) Show that the set of all positive rational numbers under the composition defined by $a * b = \frac{ab}{3}$ forms an infinite abelian group. 6,4

II. (a) Is R^2 a subspace of R^3 ? Justify.

(b) Prove that union of two subspaces W_1 and W_2 is a subspace of vector space V(F) iff they are comparable. 4,6

III. (a) If S is any subset of a vector space V(F), then show that S is a subspace if and only if L(S) = S.

(b) Find condition on a, b, c so that the vector $v = (a, b, c) \in R^3$ belongs to the space generated by u = (2, 1, 0), v = (0, 3, –4), w = (1, –1, 2). 5,5

IV. (a) If V(F) is a vector space then prove that the set S of non zero vectors $v_1, v_2, ..., v_m \in V$ is linearly dependent if and only if some vector $v_m \in S$, $2 \leq m \leq n$ can be expressed as a linear combination of its preceding vectors.

(b) Give examples of two different basis of $V_3(R)$. 8,2

V. (a) Find basis and dimension of the subspace W generated by the vectors (1, 2, 3, 5), (2, 3, 5, 8), (3, 4, 7, 11), (1, 1, 2, 3) of R^4. Also extend to a basis of R^4.

(b) Let W be a subspace of a finite dimensional vector space V(F). Then prove that :

$$\dim V/W = \dim V - \dim W.$$ 4,6

SECTION—B

VI. (a) Give an example of a linear transformation $T : U \to U$ whose range and null space are identical.

(b) Verify rank-nullity theorem for $T : R^3 \to R^3$ defined by T(x, y, z) = (x + 2y, y – z, x + 2z). 5,5

VII. Let V and W be two finite dimensional vector spaces over field F and L(V, W) be the vector space of all linear transformations from V to W. Prove that L(V, W) is also finite dimensional and dim L(V, W) = dim V . dim W. 10

VIII. (a) Show that linear transformation $T : R^3 \rightarrow R^3$ defined by $T(e_1) = e_1 - e_2$, $T(e_2) = 2e_2 + e_3$, $T(e_3) = e_1 + e_2 + e_3$, where $\{e_1, e_2, e_3\}$ is a standard basis of R^3, is neither one-one nor onto.

(b) Give an example of a linear operator $T : R^3 \rightarrow R^3$ such that $T \neq 0$, $T^2 = 0$ but $T^3 = 0$. 5,5

IX. (a) Let $T : V \rightarrow V$ be a linear operator, where V is a finite dimensional vector space over field F. Suppose $B = \{v_1, v_2, ..., v_n\}$ is a basis of V(F). Prove that $[T; B] [v; B] = [T(v); B]$ for any vector $v \in V$.

(b) Let $T : R^3 \rightarrow R^3$ be the linear transformation defined by $T(x, y, z) = (2x, 4y, 5z)$. Find matrix of T with respect to basis $\left(\frac{2}{3}, 00\right), \left(0, \frac{1}{2}, 0\right)$ and $\left(0, 0, \frac{1}{4}\right)$ of R^3. 7,3

X. (a) Find linear mapping $T : R^2 \rightarrow R^3$ determined by the matrix

$$A = \begin{bmatrix} 0 & 2 \\ 1 & -1 \\ 2 & 3 \end{bmatrix}$$

with respect to ordered basis $B_1 = \{(1, 2), (0, 3)\}$ and $\{(1, 1, 0), (0, 1, 1), (1, 1, 1)\}$ for R^2 and R^3 respectively.

(b) Let $B_1 = \{v_1, v_2, ..., v_n\}$ and $B_2 = \{w_1, w_2, ..., w_n\}$ be two ordered basis of V(F). Show that the transition matrix from basis B_1 to B_2 is invertible. 5,5